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On the generality of discontinuous fatigue crack growth in glassy polymers

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Fatigue fracture surface characteristics of five commercially available amorphous polymers [poly(methylmethacrylate) (PMMA), polycarbonate (PC), poly(vinyl chloride) (PVC), polystyrene (PS), and polysulphone (PSF)] as well as bulk-polymerized PMMA prepared over a wide range of molecular weights were studied to determine if common mechanisms of fatigue crack propagation prevail among these glassy polymers. In those polymers with viscosity-average molecular weight $\bar{M}_v \lesssim 2 \times 10^5$, the macroscopic appearance of the fracture surface showed the presence of a highly reflective mirror-like region which formed at low values of stress intensity and high cyclic test frequencies (~ 100 Hz). The microscopic appearance of this region revealed that many parallel bands exist oriented perpendicular to the direction of crack growth and that the bands increase in size with ΔK. In all instances, the crack front advanced discontinuously in increments equal to the band width after remaining stationary for hundreds of fatigue cycles. Electron fractographic studies verified the discontinuous nature of crack extension through a craze which developed continuously with the load fluctuations. By equating the band size to the Dugdale plastic zone dimension ahead of the crack, a relatively constant yield strength was inferred which agreed well with reported craze stress values for each material. At higher stress intensity levels in all polymers and all values of $\bar{M}_v$, another series of parallel bands were observed. These were also oriented perpendicular to the direction of crack growth and likewise increased in size with the range in stress intensity factor, ΔK. Each band corresponded to the incremental advance of the crack during one load cycle, indicating these markings to be classical fatigue striations.

1. Introduction

Much recent work has been done in the area of fatigue crack propagation (FCP) in glassy polymers [1–9]. A large portion of this research has been devoted to the generation of crack growth rate data in different polymers using the stress intensity factor range, ΔK, as a correlation factor. From an engineering standpoint, crack growth data obtained under cyclic conditions are valuable in enabling a designer to more intelligently select a polymeric material for a given application. To understand why one polymer is more fatigue resistant than another requires additional information about the actual fracture processes in each material. One of the most useful methods of analysing these mechanisms has been the interpretation of morphological features observed in fatigue fracture surfaces with the aid of transmission and scanning electron microscopes.

Several previous fractographic studies of fatigue fracture in glassy polymers have shown the presence of many different markings [1, 3, 8–18]. The best understood of these are the parallel striations, oriented perpendicular to the direction of crack growth, which increase in size with increasing stress intensity factor range. These striations have been observed in every glassy polymer tested under fatigue loading conditions [1, 6–10]. If the striation spacing (i.e. its width) is compared with the macroscopic crack growth rate (measured along the specimen surface) and plotted against ΔK for PS and PC, good correlation is noted [1, 3]. Therefore, one fracture band must have been formed during each load cycle, thereby reflecting the total crack front advance as a result of each load excursion. Under these conditions, it has become common practice to refer to these markings as *striations.*

These fatigue striations become smaller and eventually disappear as one moves back toward the origin of crack growth (i.e. smaller crack sizes and lower ΔK levels). At very low ΔK levels, the fracture surface appearance becomes smooth and mirror-like [3, 13–17]. In this region another series of parallel markings becomes evident. Like the fatigue striations, these bands are also perpendicular to the direction of crack growth and increase in size with increasing ΔK. However, a comparison of the band spacing and the macroscopic crack growth rate reveals that each band was not formed during one load cycle but rather over hundreds of load excursions. The mechanism for the formation of these bands has been shown to involve a discontinuous crack growth process with craze formation playing an important role [3, 11, 12]. These markings have been found in poly(vinyl chloride) (PVC) [12], PS [3], and polystyrene-polyurethane (PS-PU) [18], all uncross-linked glassy polymers with a strong propensity for crazing. It has been suggested that the width of these bands represents the size of the local plastic zone ahead of the crack tip [3, 11, 12]. Employing the Dugdale [19] plastic zone model, a yield strength was inferred which agreed well with the reported craze stress for the material [3, 11]. A further discussion of this analysis will be given later.

The objective of this study was to determine if these discontinuous crack growth markings are common to other uncross-linked glassy polymers and to ascertain the effect of testing variables such as ΔK, frequency, and molecular weight (an important structural parameter), on the formation of the markings. This paper describes and discusses results of a fractographic examination of fatigue

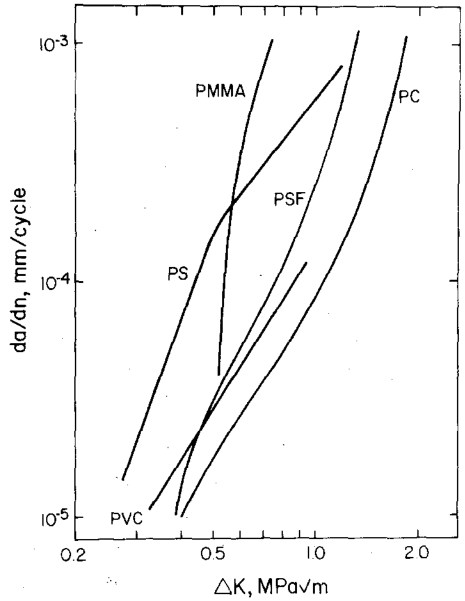


Figure 1 Relationship between crack growth rate per cycle in those polymers tested in this paper as a function of stress intensity range.

TABLE I Materials

Polymer (source)	$\bar{M}_v$	Thickness (mm)	DGB
PC			
(General Electric)	4.9×10^4	6.3	Yes
PVC			
(Cadillac)	8.8×10^4	3.2	Yes
(suspension-polymerized)	$\bar{M}_w = 0.97 - 2.3 \times 10^5$	6.4	Yes
PS			
(Cadillac)	2.7×10^5	6.5	Yes
PSF			
(Westlake)	5×10^4	4.4	Yes
PMMA			
(Rohm and Haas)	1.25×10^6	6.4	No
(bulk-polymerized cast)	$0.19 - 3.6 \times 10^6$	4 – 6.4	No
(bulk-polymerized cast)	1.1×10^5	6.2	Yes
(emulsion-polymerized)	1.0×10^5	6.4	Yes
(moulded commercial resins)	$1 - 2 \times 10^5$	5.5 – 6.4	Yes

striations and discontinuous growth bands in several polymers, in order to compare their fracture surface micromorphology.

2. Experimental procedure

Fatigue specimens used in this study were prepared from commercially available sheets of PVC, PS, polycarbonate (PC), polysulphone (PSF) and poly(methylmethacrylate) (PMMA). Other specimens of PMMA were prepared by the bulk polymerization of MMA monomer over a wide range of $\bar{M}_v$ and by the molding of several different commercial resins. The $\bar{M}_v$ of all polymers were determined by measurements of intrinsic viscosity and are given in Table I along with specimen thicknesses. FCP data for these materials were reported previously and are shown in Fig. 1. All test samples were of the compact tension geometry having $H/W = 0.6$ but with varying width. The stress intensity factor range for this specimen configuration is

$$\Delta K = \frac{Y \Delta P \sqrt{a}}{BW} \qquad (1)$$

where $Y = 29.6 - 185.5\,(a/w) + 655.7\,(a/w)^2 - 1017\,(a/w)^3 + 638.9\,(a/w)^4$, ΔP = load range, a = crack length, B = specimen thickness, W = specimen width.

Fatigue tests were conducted in a 9 kN MTS electrohydraulic closed loop testing machine at cyclic frequencies between 1 and 100 Hz. The ratio of minimum to maximum loads was held constant at 0.1 for all tests. Crack advance was measured in increments of approximately 0.25 mm with an optical travelling microscope. All specimens were tested in a laboratory air environment at room temperature.

Fractographic analysis was performed with an ETEC scanning electron microscope (SEM), a Philips EM300 transmission electron microscope (TEM), and an optical metallograph. Specimens prepared for use in the SEM were coated first with carbon and then with gold. Replicas for the TEM study were made by replicating the fracture surface with 10% aqueous poly(acrylic acid). After drying, the replica was chromium-shadowed and carbon-coated. The plastic replica was then dissolved in warm water, leaving a carbon replica for mounting on a grid for viewing.

3. Results and discussion

3.1. Macroscopic observations

At low ΔK levels in glassy polymers with $\bar{M}_v \approx 2 \times 10^5$, the characteristic macroscopic appearance of the fatigue fracture surface is smooth and mirror-like as shown in Fig. 2. A higher

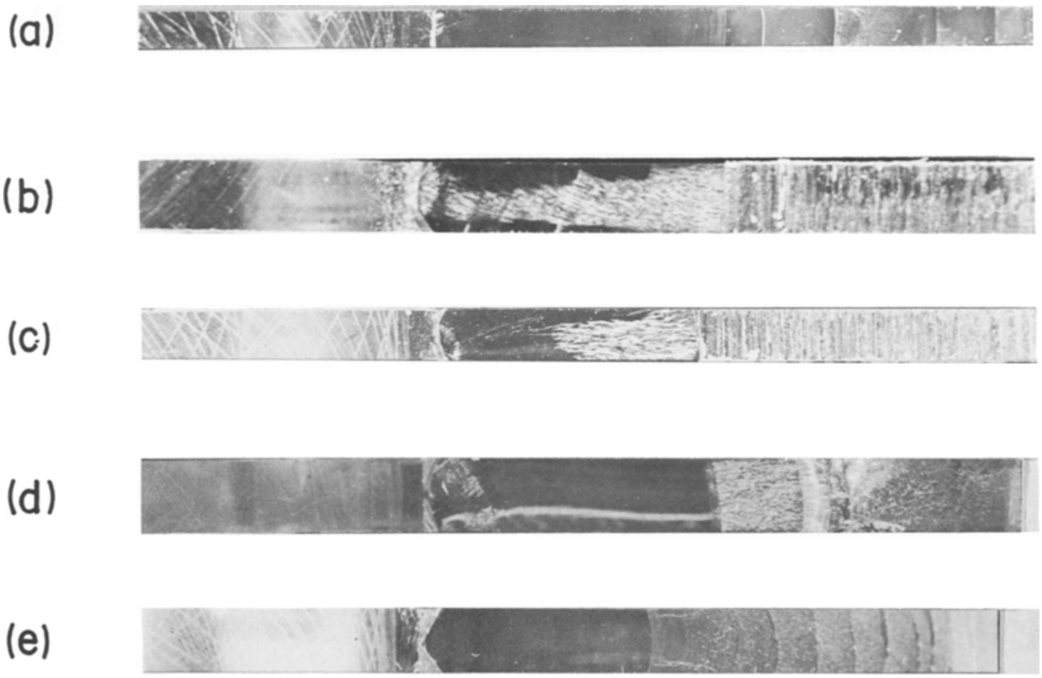


Figure 2 Fracture surfaces of (a) poly(vinyl chloride), (b) polycarbonate, (c) polysulphone, (d) polystyrene and (e) poly(methylmethacrylate).

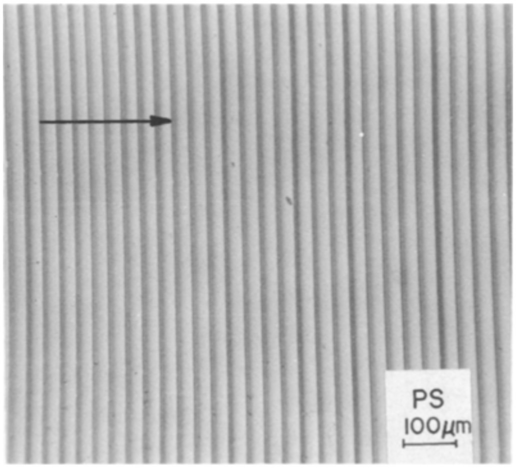


Figure 3 Parallel array of discontinuous growth bands in polystyrene. Direction of crack growth indicated by arrow.

magnification of these regions (Fig. 3) reveals hundreds of parallel bands lying perpendicular to the direction of crack growth. These bands are found in PVC, PC, PSF, PS, and low $\bar{M}_v$ PMMA (see below for discussion of effects of $\bar{M}_v$ on band formation). In each polymer, the band size increases monotonically with ΔK. From Fig. 4, it is evident that this increase follows a second power relationship for PVC, PMMA and PS throughout the ΔK range where the bands exist. For the case of PC and PSF, the data adhere to a second power correlation only over a small ΔK range. This deviation occurs simultaneously with a transition in the fracture surface appearance from a smooth to a rough texture as shown in Fig. 2b and c. During crack growth the rough region associated with multiple crazing (in which the main crack jumps from one craze to another) is observed to lag behind the smooth region as shown schematically in Fig. 5. This perturbation of the crack front profile may be responsible for hindering the development of each band and as a consequence may cause the observed complex dependence of band size on ΔK in PC and PSF.

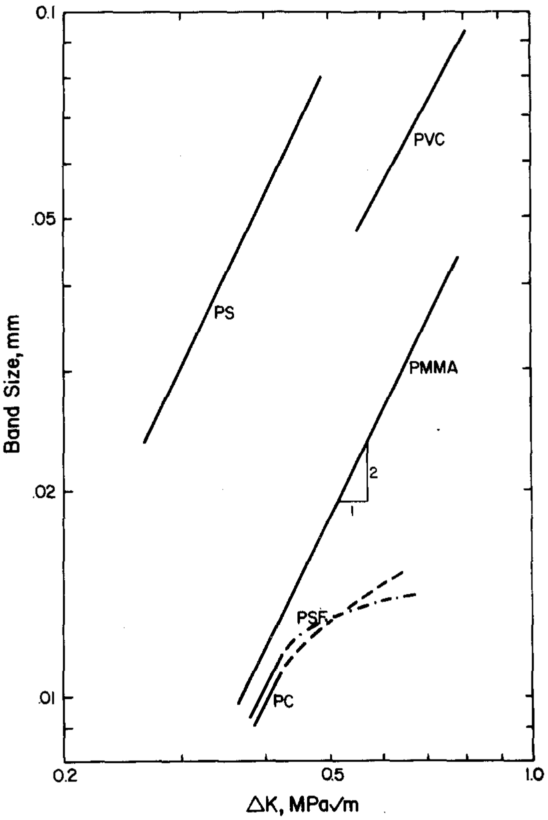


Figure 4 Dependence of band size on ΔK for five glassy polymers.

It was suggested previously in studies of PS [3] and PVC [11, 12], that the band size represents the extent of stable craze development ahead of the crack tip. Assuming the craze dimension to be equal to the crack tip plastic zone length, (r_y), one may infer an apparent yield strength (σ_y) (presumably a constant value) at different K_{max} levels through the use of the Dugdale [19] plastic strip model [20–22] which yields the following equation:

$$r_y \simeq \frac{\pi}{8}\,\frac{K_{max}^2}{\sigma_y^2}\,. \tag{2}$$

The authors used Equation 2 to compute the inferred yield strength for PS at different ΔK values [3]. This quantity was found to be relatively invariant over a considerable ΔK span with all data falling within the range of reported craze

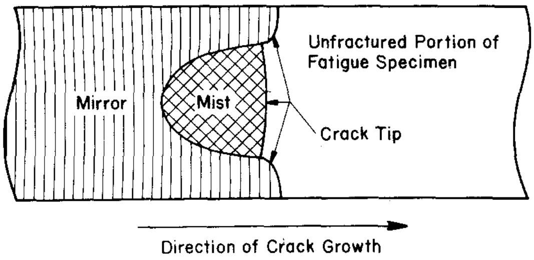


Figure 5 Schematic diagram of the crack plane during fatigue testing of PC and PSF. Note that the rough mist region associated with multiple crazing lags behind the smooth mirror region which contains the discontinuous growth bands.

TABLE II Yield strengths of polymers investigated

Polymer	Mean inferred yield strength (standard deviation) (MPa)	Reported plane strain yield strength (MPa)
PC	81 (3.6)	61 – 82 [20]
PVC		
(commercial)	51 (3.1)	47 – 65 [20]
(suspension-polymerized)	70.6 (6.4)	47 – 65 [20]
PSF	79 (1.5)	67 – 80 [20]
PS	38 (1.84)	38 [13]
PMMA		
(moulded-resin)	73 (2.9)	not available
(bulk-polymerized)	81 (0.95)	
(emulsion-polymerized)	87 (1.4)	

strength values. It is to be noted that all the inferred yield strengths were considerably below values reported for homogeneous yield strength. Since PS crazes before general yielding under tensile loading condition, this result seems reasonable. Similar measurements and calculations were made for PVC, PMMA, PC and PSF, taking band size values from the linear portions of the log–log curves in Fig. 4. These results are tabulated along with reported triaxial yield strength data in Table II. Again, good agreement is noted. On initial reflection, it is surprising to find good agreement between craze strengths obtained from a tensile test and those reported here under fatigue conditions at relatively high frequencies. It should be pointed out, however, that the fatigue crazes associated with DG band formation grow to their stable length only after many cycles. As such, the effective strain rate is actually low. The authors believe the triaxial yield strength to be a more accurate representation of the yielding process at the crack tip at low K values where the plastic zone is small and plastic constraint and tensile triaxiality are at high levels.

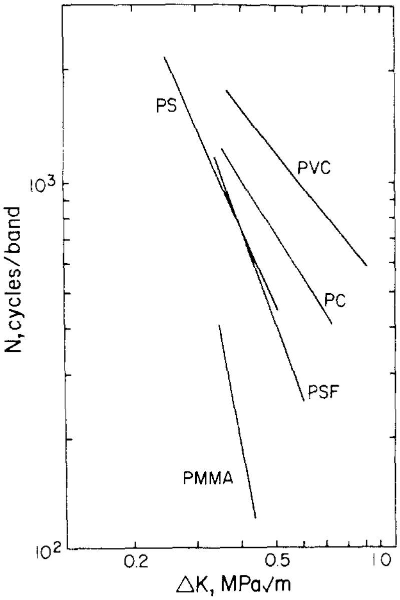


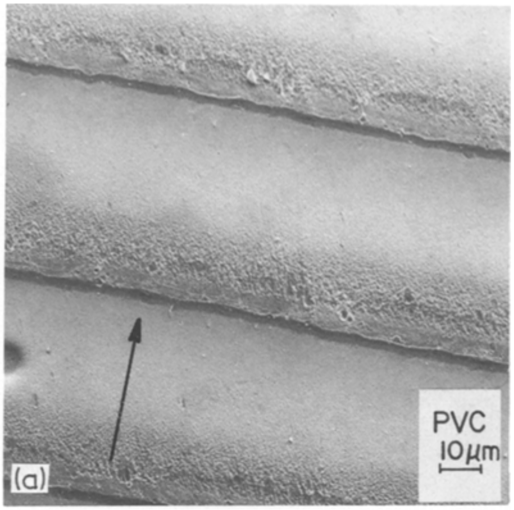
Figure 6 Effect of ΔK level on the number of cycles required for growth through a discontinuous growth band.

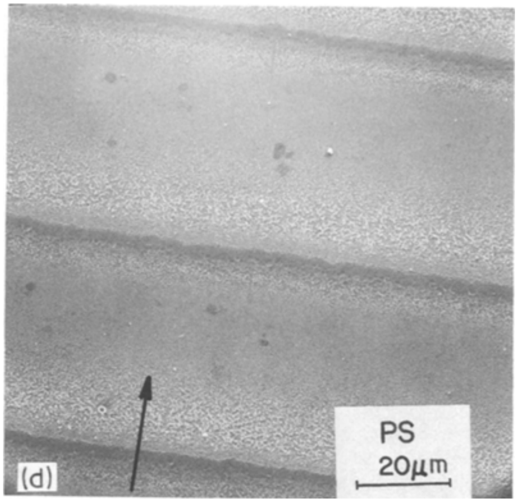
Although the macroscopic appearance of these crack growth bands are geometrically similar to fatigue striations formed during one load excursion, crack growth measurements indicate that many cycles were required to allow the crack to propagate through each band. In dividing the band width by the macroscopic FCP rate, it is seen that for the test conditions employed in this study, up to 2000 loading cycles were necessary to advance the crack through a distance equal to the band width (Fig. 6). Our initial reaction regarding this figure was to extrapolate these data to determine the ΔK level associated with 1 cycle/band (i.e. the onset of true fatigue striations). For the case of PS and PC for which striation spacing versus ΔK level values are known, the extrapolated ΔK values for these two materials were too large. That is, striations were found at lower than extrapolated values of ΔK. This discontinuity in the craze cyclic stability versus ΔK relationship is not understood at this point, nor are the third set of intermediate crack growth bands found in PS [3].

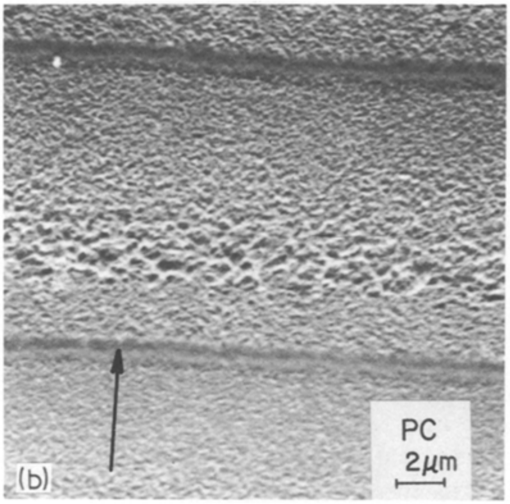
The recognition that such bands exist in these five materials bears directly on failure analyses of components made from these polymers. If these bands were misinterpreted as fatigue striations, one might conclude incorrectly that most of the fatigue life was consumed during fatigue crack initiation with only a few cycles involving FCP. In fact, this conclusion would underestimate the propagation stage of the fatigue life by more than three orders of magnitude! For this reason, it is extremely important to examine the micromorphological differences between these two markings.

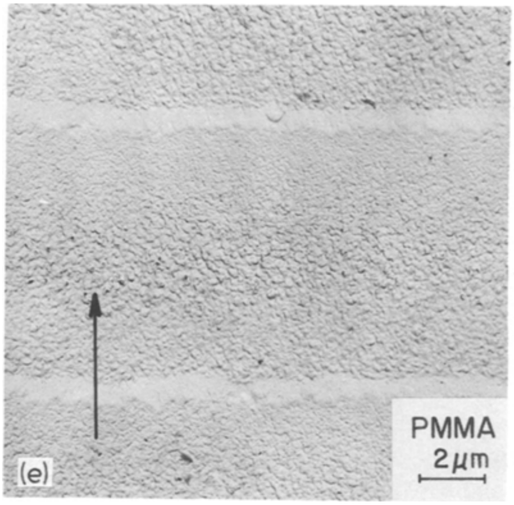
3.2. Micromorphology of discontinuous growth bands

A higher magnification of the discontinuous growth bands shows a distinctive morphology









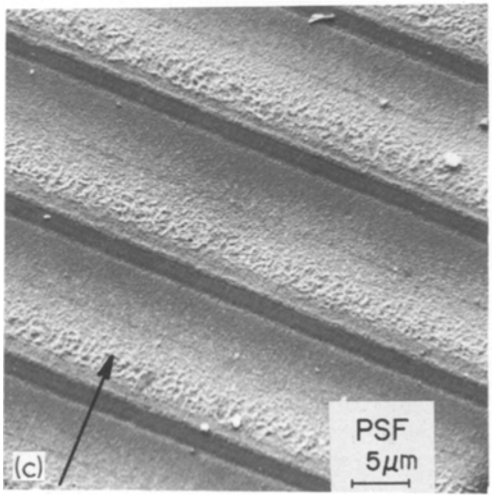


Figure 7 Fractographs of discontinuous growth bands in (a) PVC, (b) PC, (c) PSF, (d) PS (SEM micrographs) and (e) PMMA (TEM micrograph). Direction of crack propagation given by arrow.

common to all polymers tested. From Fig. 7a to e, the surface of each band is seen to contain microvoids, decreasing in size in the direction of crack growth. In a previous study of these markings in PVC, the authors [12] showed these bands to be formed by a discontinuous crack growth process shown schematically in Fig. 8a. It was concluded that under cyclic loading conditions, a single craze would grow to a stable limiting size characterized by the Dugdale plastic strip dimension. The crack was then envisioned to propagate

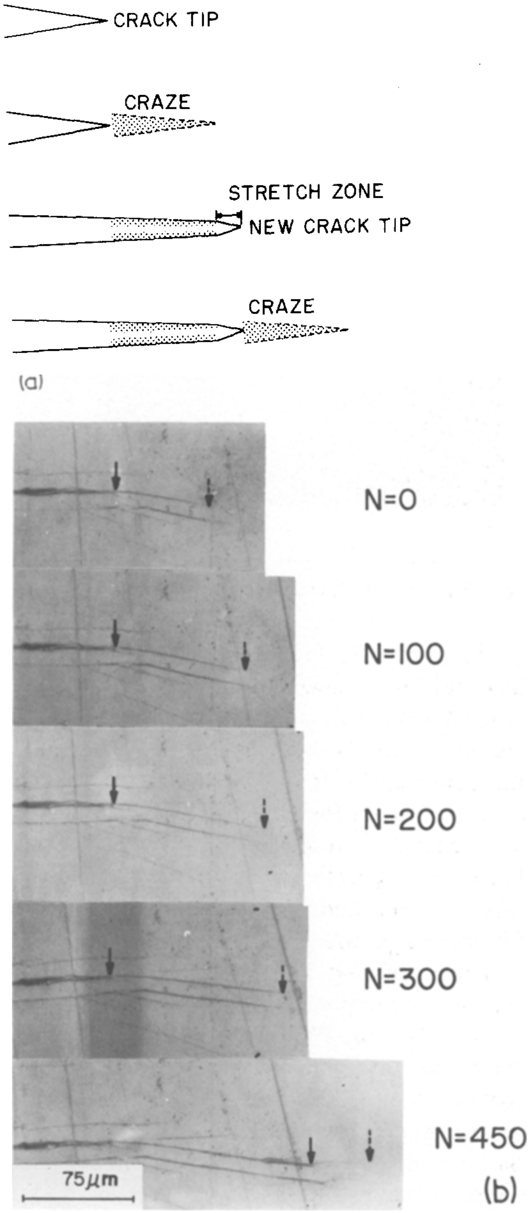


Figure 8 (a) Schematic representation of the discontinuous growth process [3]. (b) Composite micrograph of PVC showing position of craze (↓) and crack (↓) tip at given cyclic intervals.

rapidly to the tip of the weakened craze. The rapid fracturing of the craze would occur by a void coalescence mechanism with the void size distribution reflecting the internal structure of the craze just prior to crack extension, as determined by the crack opening displacement distribution at the craze tip. Blunting, represented by the stretch zone following each band, and crack arrest would occur when the crack reached homogeneous uncrazed material. This discontinuous crack growth mechanism was confirmed [12] during fatigue tests of PVC by observations of crack tip jumping, which occurred intermittently after the crack tip remained stationary for several hundred loading cycles (Fig. 8b). In low $\bar{M}_v$, plasticized PVC each jump was accompanied by a slight but audible "click" which confirmed the occurrence of a discontinuous cracking process [10].

The previous discussion provided a generalized view of the discontinuous crack growth mechanism. A more complete analysis of crack growth through a craze is now required to describe certain variations in discontinuous growth band (DGB) morphology. All polymers examined, and especially PS and PVC, showed a narrow region of coarser microvoids immediately preceding the stretch zone (i.e. at the end of the DGB). Close examination reveals this region to be comprised of intermediate sized voids. It is believed that the presence of these voids reflect changes in the rate of crack growth through the craze. The velocity of crack penetration of the initial portion of the craze is quite high due to the low energy needed to fracture a region consisting predominantly of large voids. As the crack approaches the end of the craze where the void size diminishes and the craze becomes more dense, additional energy would be required for failure. This should result in a decrease in crack velocity prior to blunting, thereby providing additional time for void coalescence at the end of each band. This region is most prominent in PVC and PS in which crazes develop rapidly.

Another aspect of DGB morphology relates to the position of the largest voids in each band which should correspond to the region of maximum tensile triaxiality. From Fig. 7, these voids were found in all polymers at distances of from $\frac{1}{6}$ to $\frac{1}{3}$ of the band width from the origin. For an infinitely sharp crack, maximum triaxiality will exist at the crack tip. Since blunting occurs after craze fracture during discontinuous crack growth, the crack tip radius is most certainly finite. From the results by Creager and Paris [23], it can be shown that the region of maximum triaxiality will move further away from the crack tip the greater the crack tip radius. Hence, the greatest void size should also occur away from the end of the stretch zone (i.e. the crack tip). While no attempts to measure crack tip radii were made in this study, it is interesting to note that the tougher

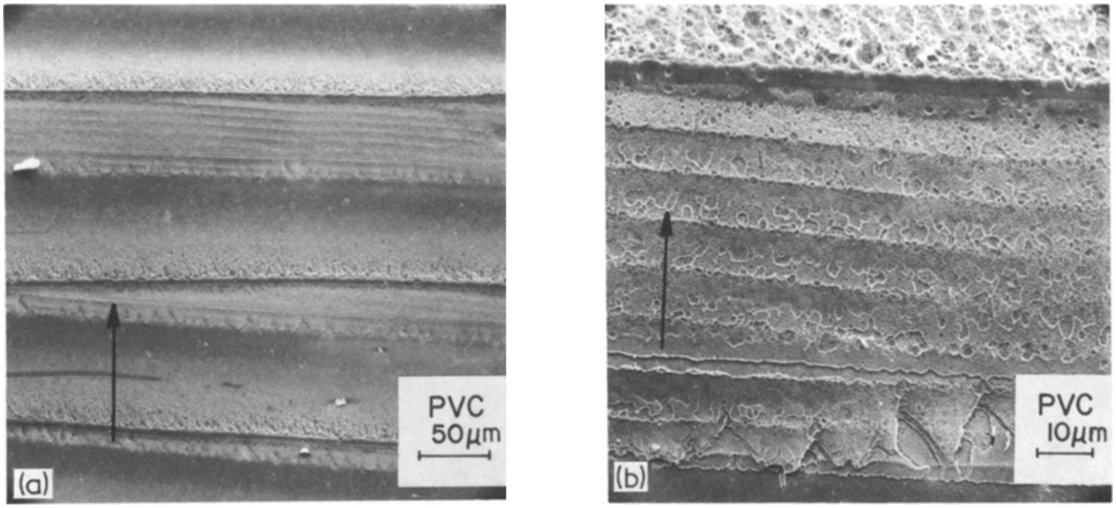


Figure 9 (a) SEM fractograph revealing presence of smaller parallel bands within bands nearest the final stable crack front location. (b) Note evidence of crack jumping between craze interfaces in the layered structure of the bands.

polymers, PSF and PC, which should exhibit appreciable blunting, showed maximum void diameters furthest from the DGB origin.

An additional variant in DGB morphology was found in PVC at high growth rates just before fracture. As shown in Fig. 9a, the last five bands contained smaller parallel markings located toward the end of each DGB. The morphology of these bands (Fig. 9b) is indicative of crack jumping from one craze interface to another. Similar fracture markings were reported by Murray and Hull [24] in rapid fracture studies of PS. This PVC fracture surface feature probably reflects the fact that the velocity of crack growth through these last few crazes was approaching conditions associated with rapid fracture. The observation of these markings only in PVC is reasonable since it is the only polymer in the group examined to sustain discontinuous crack growth until final failure.

From the previous discussion, it is apparent that crazing must play a critical role in discontinuous crack growth. Consequently it is not surprising that DG bands are found in PS, PMMA and PVC, materials which craze easily; however, PC and PSF are not observed to craze in inert environments under monotonic loading conditions in unnotched samples [25]. We attribute the presence of crazes in these materials to the large triaxial stresses at the crack tip and perhaps also to the cyclic nature of the loads.

Since FCP in the discontinuous crack growth regime depends primarily on the characteristics of the craze ahead of the crack, those materials which require a large number of cycles (N) to rupture the craze should prove to be most fatigue resistant. From Fig. 6, N is observed to decrease with increasing ΔK for all polymers tested. (Recall that striations are found at ΔK levels below those associated with extrapolated values for 1 cycle/band.) This reflects the increasing amount of damage accumulated in the craze per loading cycle with increasing ΔK level. It is apparent from a comparison of Figs. 1 and 6 that the trend in N does not correspond to the observed fatigue response. That is, PVC appears to be the most fatigue resistant polymer. This lack of correlation is believed due to the large variation in the yield strength which causes the band size to differ drastically. (Recall that the Dugdale plastic zone dimension varies with inverse square of σ_y.) If two materials could be compared with similar σ_y effect of craze stability on crack growth rates (da/dn) could be isolated. Such a comparison is made in Fig. 10. Bulk polymerized PMMA ($\bar{M}_v = 1.1 \times 10^5$) and commercial PC have approximately the same inferred σ_y (approximately 80 MPa). It is apparent that the number of cycles required to fracture the craze in PC is nearly two orders of magnitude higher than PMMA. The effect of this difference in craze stability may be seen in the FCP data in Fig. 10b where for a given ΔK, PMMA has a crack growth rate about 50 times higher than PC.

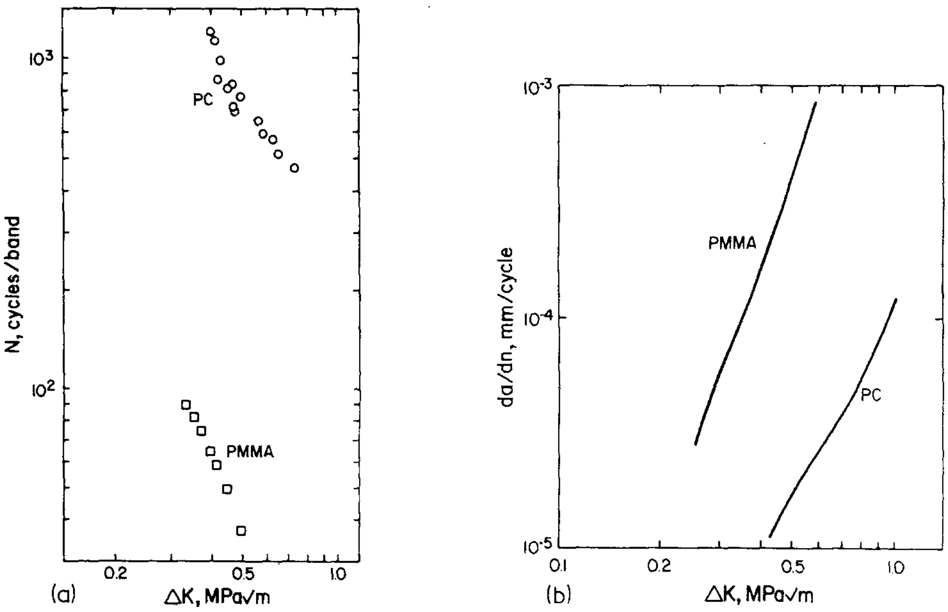


Figure 10 (a) A comparison of the number of cycles required for growth through one discontinuous growth band as a function of ΔK in PC and PMMA. (b) Relationship between crack growth rates as a function of ΔK in PMMA and PC. (Both have the same inferred yield strength.)

3.3. Fatigue striation morphology

It is appropriate in this study of discontinuous growth morphology to also discuss the structural appearance of fatigue striations which are formed at higher growth rates during one loading cycle. In this manner, the reader could make a direct micromorphological comparison between DG bands and striations. Of all the polymers tested only PVC did not exhibit true fatigue striations under any conditions; instead PVC showed DG bands over the entire fatigue fracture surface. Fractographs of fatigue striations in all the other polymers (Fig. 11) showed remarkably similar appearances. The striations were very flat with a fine linear structure (oriented parallel to the crack propagation direction). In general, the arrest line between striations was very narrow. Careful observation of either side of the arrest line shows no significant difference in texture as had been noted by Jacoby [8]. If one were to compare these fatigue striations with the DG bands shown in Fig. 7, the difference in morphology becomes quite evident. In fatigue fracture analysis of polymers where bands are visible, but growth rate data are unavailable, attention should be given to band micromorphology in order not to misinterpret the particular mechanism for band formation.

3.4. Effect of ΔK, frequency, and M on DGB formation

Discontinuous growth bands are found to exist near the origin of crack growth where short single crazes are known to precede crack growth. In a majority of the polymers tested, the bands disappear at the point where multiple crazing begins. This suggests that DG bands are found where ΔK levels are low enough to prevent the development of craze bundles which fragment the crack front.

The effect of frequency on discontinuous crack growth is not the same in all polymers tested. In PVC and low $\bar{M}_v$ PMMA, DG bands have been found at all test frequencies while PS, PC, and PSF require high frequencies (100 Hz) for their formation. The need for high frequency in some polymers may be traced to the effect of frequency on craze formation and the chain segment jump frequency responsible for the β transition [26]. At high loading rates which occur during rapid cycling, little time is available for the initiation and growth of many crazes. As a consequence, the region associated with single craze development is stabilized relative to the formation of craze bundles. This is favourable to the prolonged stability of DG band formation.

Although the effect of M on DGB development

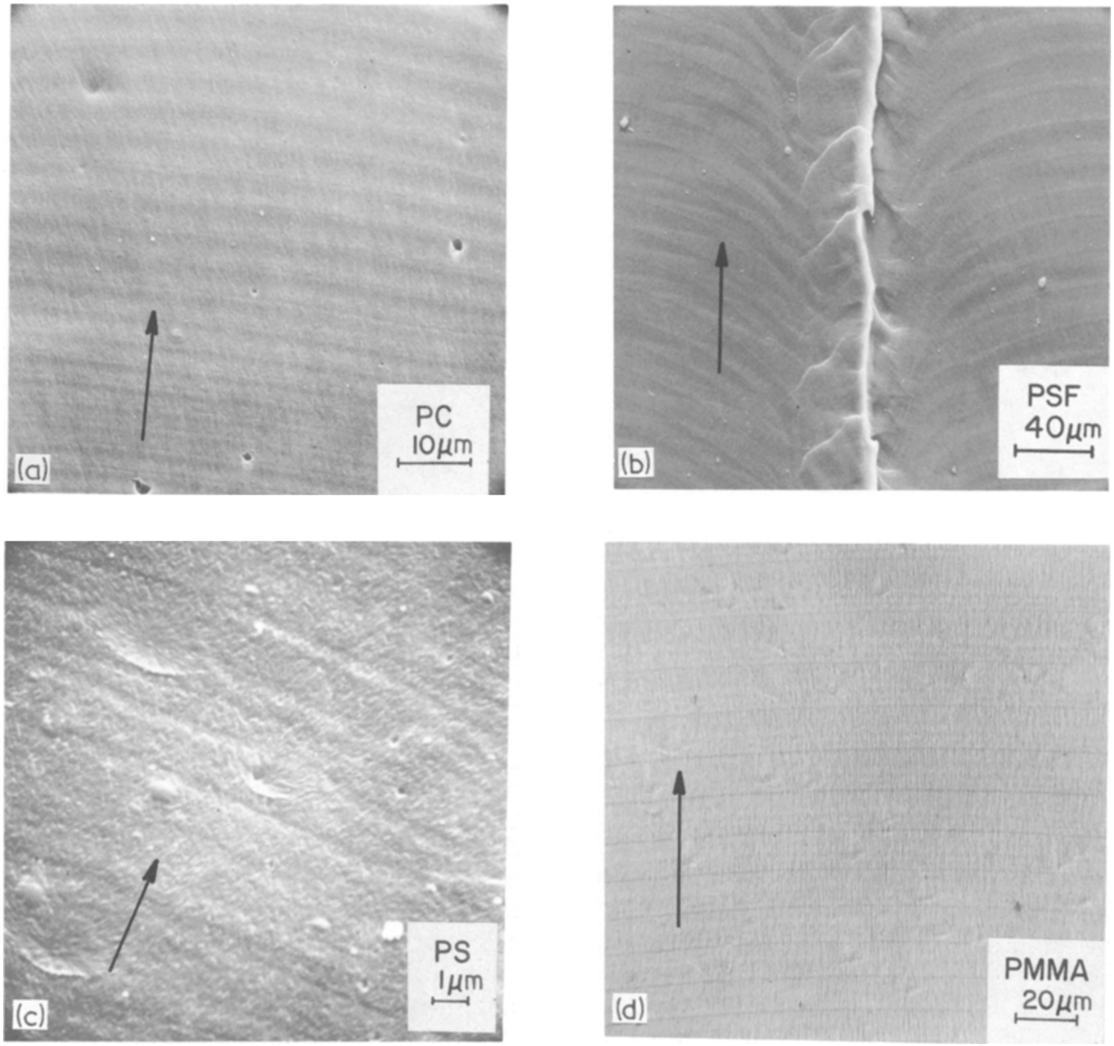


Figure 11 Typical striation morphology in (a), PC, (b) PSF, (c) PS (SEM micrographs), and (d) PMMA (light micrograph). Arrow indicates direction of crack growth.

is not clearly understood at this time, a possible trend may be seen. From Table I, all polymers examined in this study which exhibited DG bands had $\bar{M}_v \lesssim 2 \times 10^5$. No DG bands were visible in pure bulk-polymerized (cast) PMMA over a range of $\bar{M}_v = 3 \times 10^6$ to 1.9×10^5 nor were any visible in commercially cast PMMA where $\bar{M}_v = 1.6 \times 10^6$. However, in bulk- and emulsion-polymerized PMMA, moulded commercial PMMA resins, and all other polymers with $\bar{M}_v \lesssim 2 \times 10^5$, DG bands were visible.

A molecular weight of about 10^5 or 2×10^5 may represent a critical chain length for craze formation. For low M polystyrene, Fellers and Kee [27] showed crazes to be short and weak. A similar craze weakness at low M was also reported by Kambour [28]. The cyclic deformations associated with dynamic loading may induce the short molecular chains to disentangle causing failure of the craze to occur at relatively low loads and over a small number of fatigue cycles. At higher M, the molecular chains are longer and chain entanglements more effective in resisting chain sliding during fatigue. The crazes which form in high M materials may well be stronger, thus requiring a greater driving force (higher ΔK) for failure to occur. Such materials are also seen to form craze bundles readily [26]. All these characteristics of crazes at high M will be unfavourable to DGB formation, and may account for the absence of DG bands at values of M above approximately 10^5. Although specimens of PSF, PC, PVC,

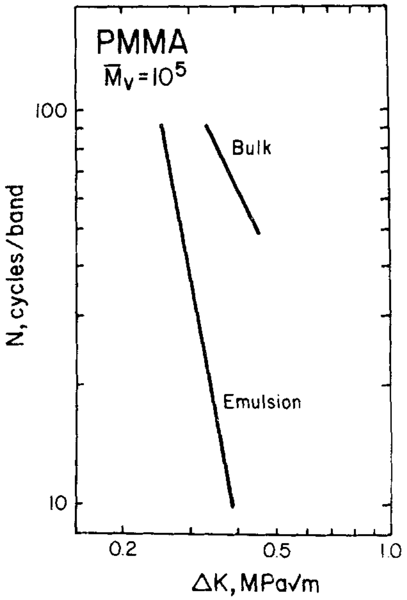


Figure 12 Effect of $\bar{M}_w$ on the number of cycles required to fracture a DG band in PVC as a function of ΔK.

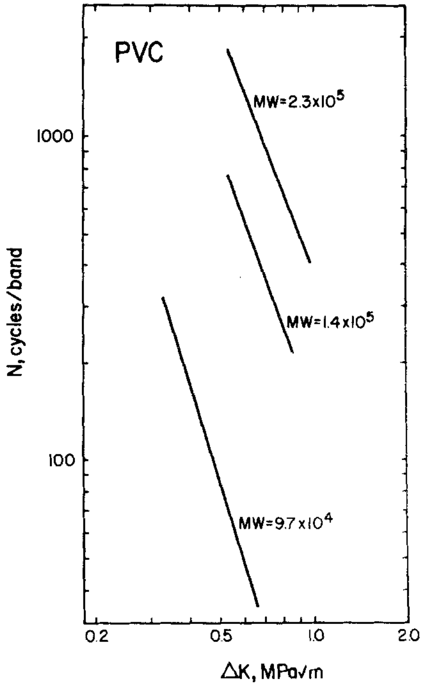


Figure 13 Effect of MWD on the number of cycles needed for crack growth through one DG band in PMMA as a function of ΔK ($MWD_{bulk} > MWD_{emulsion}$).

and PS with $\bar{M}_v \lesssim 2 \times 10^5$ were not available, one might suspect such materials to be more resistant to DG band formation.

The simplistic chain disentanglement model offered above can be further developed by utilizing an approach presented originally by Berry [29] and later, in more detail by Kusy and Turner [30]. They suggested that craze stability could be maximized only if the molecular chains, when extended, were long enough to be anchored at each internal face of a craze. However, Kausch [31] has shown that complete molecular extension is physically unrealistic. In the unstrained condition, the amorphous structure is assumed to consist mainly of interpenetrating coils of these stretched coils. The coils are believed to be extended by only a factor of 2 or less. In any case, it is conceivable that under cyclic loading, the interpenetrating coils begin to slip past each other so that the total number of chains entangled in the fibrils decreases. Since the size of the coil varies with $\sqrt{M}$, coils of a high-M polymer will intrinsically have a more dense network of entanglements than coils of a low-M polymer. Consequently, after a given number of cycles at a particular ΔK level, the material with the higher M will retain a more effective entanglement network than a polymer with a lower M. As a result, the fibrils within the craze of the higher M material should be stronger and hence, more resistant to fracture by discrete bursts across the craze's entire area. Above some limiting value of M ($\sim 2 \times 10^5$ from our experimental data), it is suggested that slow slippage of the coils occurs instead and results in continuous FCP. This hypothesis is supported by results given in Fig. 12 showing the crase stability in PVC to decrease with decreasing M [32]. Conversely, the comparative strength of fibrils in the low-M polymer will be quite small. Therefore, when crack growth through a craze is initiated by the failure of a few fibrils near the crack tip, the remaining fibrails are too weak to restrain the continued advance of the crack. This condition promotes discontinuous crack growth.

Finally, this model predicts (as implicit in the arguments of Berry [29] and Kusy and Turner [30] that craze stability should increase with an increase in the proportion of higher M material in a given molecular weight distribution since there would then be a higher effective entanglement density per fibril [34]. Additional results reported elsewhere [34] show that for the same $\bar{M}_v$, emulsion polymerized (with a presumably narrow MWD) PMMA exhibited higher FCP rates than bulk polymerized (whose MWD is skewed to higher M)

PMMA. Fig. 13 reveals a greater craze cyclic stability in the latter material – a fact which may be explained by the model.

4. Conclusions

From observations of discontinuous growth bands formed on the fracture surfaces of several polymers, one can conclude that discontinuous crack growth is an FCP mechanism common to uncrosslinked glassy polymers. Discontinuous growth band formation is favoured by the development and growth of short single crazes ahead of the crack tip. These conditions are met at low ΔK and high frequency. Molecular weights below about 2×10^5 also seem necessary for producing a small number of weaker crazes which encourage discontinuous crack growth. By equating band length to the computed plastic zone size dimension, inferred yield strengths were calculated which agreed well with reported values of craze and triaxial yield stresses. At high ΔK levels, continuous crack growth was made evident by the presence of fatigue striations corresponding to the advance of the crack during each loading cycle.

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